EXERCISE SOLUTION FOR WTR

Case			
base	K = 1x10^{-2} m/day Precipitation = 500 mm/yr Runoff = 0 mm/yr ET = 400 mm/yr		
	Variable	Value	Comments
	volumetric recharge	1.37x10^{-2} m^3/day/m	The volume of recharge is the product of the recharge rate (i.e., precipitation rate – runoff rate - evapotranspiration rate) and the surface area which is 50 m long and 1 m wide.
	d, divide	18.2 m	The divide is close to the river which is reasonable given its higher stage.
	h_{max}	5.9 m	The head is below maximum surface elevation of 6 m. However, if the surface is lower than 6 m at x=18.2 m then there would be seepage at the ground surface.
	$h_{12.5}$	5.8 m	The head is below maximum surface elevation of 6 m. However, if the surface is lower than 6 m at x=12.5 m then there would be seepage at the ground surface.
	$h_{37.5}$	5.0 m	The head is below maximum surface elevation of 6 m. However, if the surface is lower than 6 m at x=37.5 m then there would be seepage at the ground surface.
	q_{river}	-5.0x10^{-3} m^3/day/m	The negative value reflects flow in the negative x direction, which is flow to the left, into the river. This is approximately 1/3 of the recharge, which makes sense because the divide is 1/3 of the distance between the river and the canal.
	q_{canal}	+8.7x10^{-3} m^3/day/m	The positive value reflects flow in the positive x direction, which is flow to the right, into the canal. This is approximately 2/3 of the recharge, which makes sense because the distance form the divide to the canal is 2/3 of the distance between the river and the canal. The sum of the outflow to the river and the canal equals the recharge to the surface, thus the volumetric budget balances.
a)	base case except K = 1x10^{-4} m/day		
	Variable	Value	Comments
	volumetric recharge	1.37x10^{-2} m^3/day/m	The volume of recharge is the product of the recharge rate (i.e., precipitation rate – runoff rate - evapotranspiration rate) and the surface area which is 50 m long and 1 m wide.
	d, divide	24.9 m	The divide is close to the middle indicating that the recharge has greater influence on the shape of the water table than the stage of the river and canal.
	h_{max}	41.6 m	The head is above surface elevation of 6 m. Heads cannot be above the ground surface in an unconfined aquifer because groundwater would discharge at the surface, lowering the head. In this case, we conclude the recharge rate or the hydraulic conductivity are not representative of the field conditions because the heads of the boundary features are easy to confirm in the field. Thus, we would adjust one or both of R and K until the simulated heads match the heads observed in the field. That process is called calibration.
	$h_{12.5}$	36.1 m	Again, heads are above surface elevation of 6 m
	$h_{37.5}$	36.0 m	Again, heads are above the ground surface. Also of note is that the head distribution is nearly symmetrical because the recharge flow is dominating over the flow through the aquifer from the river to the canal.
	q_{river}	-6.8x10^{-3} m^3/day/m	The negative value reflects flow in the negative x direction, which is flow out to the river. This is roughly half of the recharge.
	q_{canal}	+6.9x10^{-3} m^3/day/m	The positive value reflects flow in the positive x direction, or outflow to the canal. The sum of the outflow to the river and the canal equals the recharge to the surface, thus the volumetric budget balances.

b)	base case except K = $1x10^{-0}$ m/day		
	Variable	**Value**	**Comments**
	volumetric recharge	$1.37x10^{-2}$ m^3/day/m	The volume of recharge is the product of the recharge rate (i.e., precipitation rate – runoff rate - evapotranspiration rate) and the surface area which is 50 m long and 1 m wide.
	d, divide	(-659) m so 0 m	Water flows in from the left side and out the right so the calculated "divide" is negative reflecting the distance to the left that would be needed to collect recharge at the rate R in order to equal the inflow from river. That is, it would take 659 m of land receiving recharge of $2.7397x10^{-4}$ m/day to yield an inflow of 0.1803 m^2/day from the river (shown below as q_{river}).
	h_{max}	12 m	This is the head that would occur at -658 m if material of K = 1x10-1 m/day extended that far.
	$h_{12.5}$	4.6 m	This is less than head at the left boundary which is consistent with inflow from the river.
	$h_{37.5}$	3.5 m	This value falls between those to the left and the right which is consistent with all flow in the system going from left to right.
	q_{river}	+0.1803 m^3/day/m	This is substantially larger than the recharge to the system indicating that flow through from one fixed head boundary to the other dominates over inflow from recharge. The river flow should be checked to determine if there is enough water flowing by in the river to provide this inflow for every meter along the rivers path.
	q_{canal}	+0.194 m^3/day/m	This equals the sum of the inflow at the river and the recharge to the surface ($1.37x10^{-2}$ m^3/day/m), thus the volumetric budget balances.
c)	base case (K = $1x10^{-2}$ m/day) except ET = 550 mm/yr		
	Variable	**Value**	**Comments**
	volumetric recharge	$-6.84x10^{-3}$ m^3/day/m	The volume of recharge is the product of the recharge rate (i.e., precipitation rate – runoff rate - evapotranspiration rate) and the surface area which is 50 m long and 1 m wide. It is negative in this case because ET is larger than precipitation minus runoff.
	d, divide	38.7 m	The divide is closer to the right side of the land mass because the flow pattern is inverted given the recharge is negative given the high value of ET, resulting in net outflow to the surface
	h_{max}	2.35 m	This head is lower than both boundary heads because the negative recharge causes a concave water table surface.
	$h_{12.5}$	3.86 m	This is less than head at the left boundary which is consistent with inflow from the river and is higher than the head at the divide which is consistent with net outflow to the surface.
	$h_{37.5}$	2.36 m	This is less than the boundary head which is consistent with inflow from the canal and higher than the head at the divide which is consistent with net outflow to the surface.
	q_{river}	$+5.3x10^{-3}$ m^3/day/m	The positive value reflects inflow from the river. The value is less than the total loss due to the negative recharge.
	q_{canal}	$-1.55x10^{-3}$ m^3/day/m	The negative reflects inflow from the canal. When combined with inflow from the river this balances the outflow (net loss to evapotranspiration $-6.84x10^{-3}m^3$/day/m) to the surface, thus the volumetric budget balances.

d) **base case ($K = 1x10^{-2}$ m/day) except ET = 500 mm/yr**

	Variable	Value	Comments
	volumetric recharge	0 m^3/day/m	There is nor recharge because precipitation is exactly offset by runoff and ET.
	d, divide	-INFINITY	Given zero recharge an infinite collection area would be required to the left of the model to satisfy the inflow from the river.
	h_{max}	5.1 m	The maximum head will occur at the left boundary.
	$h_{12.5}$	4.6 m	This is less than head at the left boundary which is consistent with inflow from the river and greater than the head at the canal which is consistent with flow from left to right through the aquifer.
	$h_{37.5}$	3.5 m	This is less than head at the left boundary which is consistent with inflow from the river and greater than the head at the canal which is consistent with flow from left to right through the aquifer.
	q_{river}	$+1.87x10^{-3}$ m^3/day/m	The positive value reflects inflow from the river.
	q_{canal}	$+1.87x10^{-3}$ m^3/day/m	The positive value reflects outflow to the canal. It equals the inflow from the river because there is no additional recharge from the surface, thus the volumetric budget balances.